The kinetic theory of gases

(an important basic equation)

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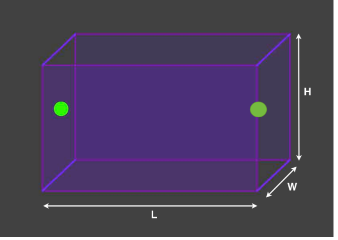
1



Imagine a single molecule in a box rebounding from one end to another

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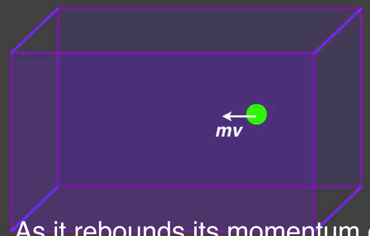

Total change in momentum is:
mv-(-mv)
which is:
2mv

Assuming the collision is perfectly elastic

As it rebounds its momentum changes from one direction to the opposite one

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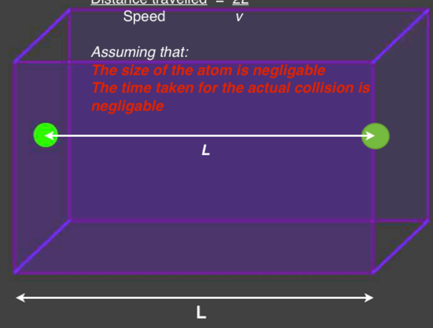
The time between collisions is:
$\frac{\text{Distance travelled}}{\text{Speed}} = \frac{2L}{v}$

Assuming that:
The size of the atom is negligable
The time taken for the actual collision is negligable



To calculate the average force on that face we need the rate of change of momentum

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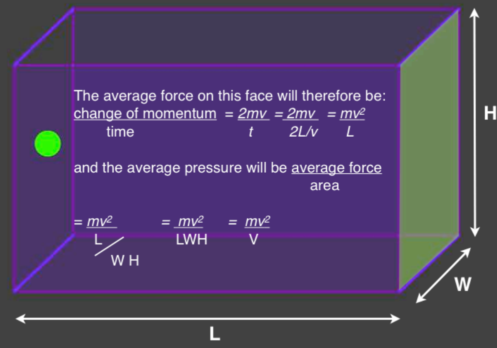

The average force on this face will therefore be:
$\frac{\text{change of momentum}}{\text{time}} = \frac{2mv}{t} = \frac{2mv}{2L/v} = \frac{mv^2}{L}$

and the average pressure will be $\frac{\text{average force}}{\text{area}}$

$= \frac{mv^2/L}{W\,H} = \frac{mv^2}{LWH} = \frac{mv^2}{V}$

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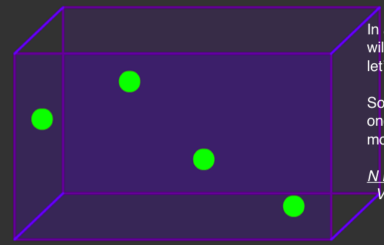

In a real box of gas there will be many molecules, let's call that N

So the average force on one face due to N molecules is:

$\frac{N\,mv^2}{V}$

but the molecules to not just travel right to left but in all three dimentions and we assume that there are so many molecules and that there motion is random that the vector sum of there velocities in any one direction is the same as in any other.

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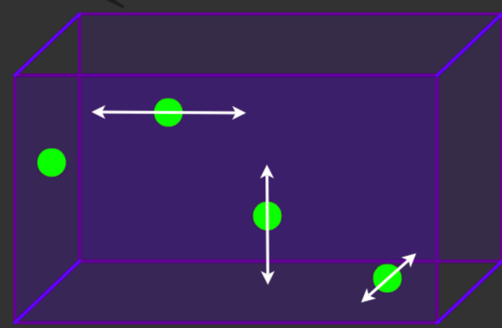

So the average force on one face is:

$\frac{N\,mv^2/V}{3}$

$= \frac{N\,mv^2}{3V}$

We are assuming that:
there are very large numbers of molecules

that their motion is random

and we are assuming that there are no forces between molecules

We are assuming that no forces between the molecules disturb the uniform velocity between collisions

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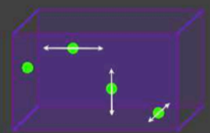
We can arrange the equation in different ways



So we now have an expression for the average pressure in the box as:

$P = \frac{N\,mv^2}{3V}$

but the molecules all have different velocities and for every one going up there will be one going down, for everyone travelling left there will be one travelling right and because velocity is a vector the average is zero.

This makes no sense for this analysis so rather than use average velocity we employ the concept of the root mean square velocity.

The root mean square velocity or rms value is calculated by squaring every value (so they are all positive) finding the average and then taking the square root.

The symbol for the rms velocity is $\bar{c}$ so now the equation becomes:

$$P = \frac{N\,m\bar{c}^2}{3V}$$

this is often rearranged a little to give $PV = \frac{1}{3} N\,m\bar{c}^2$

or like this $P = \frac{1}{3}\frac{N\,m\bar{c}^2}{V}$ which gives $\frac{1}{3}\rho\,\bar{c}^2$

where ρ is the density of the gas

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A knowledge and understanding of the assumptions made about an ideal gas is often required in examination

In this argument the following assumptions have been made:

- **The collisions are perfectly elastic**
- **The size of the molecules is negligable compared to the volume they occupy**
- **There are very large numbers of molecules**
- **The motion of the molecules is random**
- **The time taken for the actual collision is negligable compared to the time between collisions**
- **There are no forces between molecules except at the instant of collision**

These are the assumptions of an "Ideal Gas".
They are reasonable for most common light gases at low pressures but not for those with larger molecules or for gases at high pressure.